# A three-dimensional computational fluid dynamics model of a rocket

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**Abstract:** This paper proposes a 3D computational fluid dynamics (CFD) model of a rocket to visualise the oblique shock-wave cone at a certain Mach number. Its angle is then compared to the angle of a real such cone on a single still film frame to verify the accuracy of the goniometric algorithm for rocket velocity computation from this frame. The Mach number is adjusted iteratively to minimise the difference. This model of the *Saturn V* rocket at *S-IC* staging time verifies the Mach number computation method defined earlier.

**Триизмерен аеродинамичен изчислителен модел на ракета**

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**Резюме:** В статията се предлага триизмерен аеродинамичен изчислителен модел на ракета с цел изобразяване на конуса на ко́сата ударна вълна при дадено число на Мах. Неговият ъгъл се сравнява с ъгъла на реалния конус от филмов стоп-кадър за проверка на точността на гониометричния алгоритъм за изчисляване на скоростта на ракетата по този кадър. Числото на Мах се коригира итеративно, за да се получи възможно най-малка разлика. Този модел на ракетата „Сатурн-5“ при отделяне на степен „*S-IC*“ проверява метода за изчисляване на числото ѝ на Мах, дефиниран по-рано.

## 1. Introduction

Above a certain supersonic Mach number, a conical shock wave is attached to the aircraft cone's apex. The wave's cone angle is often used to determine the Mach number [1] (in wind tunnels to test airfoils, engines, etc.). Pokrovsky in 2007–2010 and Popov in 2010 did this to a rocket in open air [4], analysing NASA's *Apollo 11* launch clip [5] and its still frame [6] by this and three other methods. An author's paper defined a rocket velocity computation formula and algorithm to get a more accurate rocket speed value [7]. This paper aims at verifying the formulae, the algorithm, and the Mach number obtained through it by modelling the supersonic airflow past the rocket at this Mach number and comparing the angle of the so obtained oblique shock-wave to the measured one.

## 2. Model creation steps

The most widely used CFD software, *Fluent*®, was selected as simulation "workhorse". The following objects were created in its geometric modelling and mesh-generation software, *Gambit*®:

1. A real size (±1 *mm*), simplified 3D drawing of the *Saturn V* rocket at separation time.
2. A parallelepiped "brick" volume for the airflow around the rocket and connected to it.
3. A triangular element surface mesh for the launch-escape motor with a spacing of 0,2 *m*.
4. A triangular element surface mesh for the rest of the rocket with a spacing of 0,47 *m*.
5. A tetrahedral element volume mesh for the modelled airflow with a spacing of 2 *m*.
6. A surface group named "rocket" uniting all the rocket surfaces.
7. A surface group named "farfield" uniting all the "brick" surfaces.
8. A boundary zone of type *WALL* named "rocket" – from the "rocket" group.
9. A boundary zone of type *PRESSURE_FAR_FIELD* named "farfield" – from that group.

A mesh file was exported to *Fluent* and imported there. The following operations were done there:

A. Scaling of the imported grid 1:1000 to be in metres (was defined in millimetres in *Gambit*).
B. Reordering the grid (both the domain and zones) to increase the memory access efficiency.
C. Selecting the explicit density-based solver as the flow is supersonic and thus compressible.

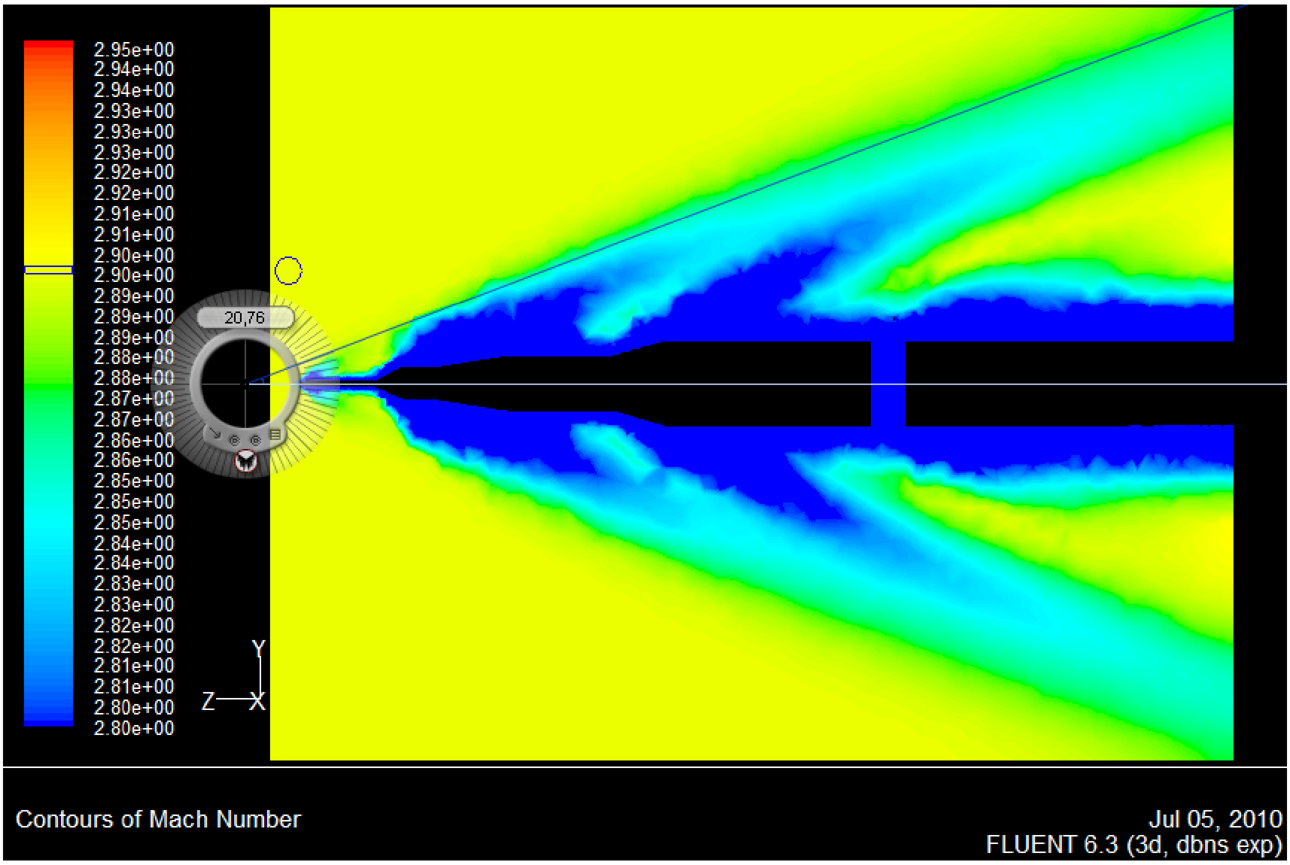


**Fig. 1:** Cross-Z-axis section of 3D *Saturn V* rocket shock wave cone model at Mach 2.9.
*Solver acronyms:* "dbns" = Density-Based Navier-Stokes, "exp" = explicit, "imp" = implicit.

D. Selecting the Green-Gauss node-based gradient option for better accuracy.
E. Setting the viscous model to *inviscid*.
F. Enabling the energy equation for the model.
G. Setting the fluid material to *air* and density to *ideal gas*.
H. Setting operating pressure condition to 0 to make absolute pressure equal to gauge pressure
I. Setting the "farfield" boundary condition to 270.65 *K*, 101.325 *Pa* and Mach 2.9 [7].
J. Setting the *X*- and *Y*-components of flow direction to 0, and the *Z*-component to −1.
K. Setting the minimum absolute pressure solution limit to 0.1 *Pa*.
L. Setting the following flow equation solution controls: flow discretisation = *first order upwind*, Courant number = 2, residual smoothing iterations = 2, smoothing factor = 0.5.
M. Computing the solution initialisation from the "farfield" zone.
N. Initialisation of the solution.
O. Enabling the *plot* option of the residual monitors.
P. Doing 1000 iterations. The solution converged at iteration 821 in ≈80 *min* (*Core Duo/2GB*)

The computed Mach 2.863 [7] resulted in a slightly greater angle. A second iteration at Mach 2.9 gave the 20.76° angle measured in [7] (fig. 1). So, the computed Mach number error is just 1%.

## 3. Remarks

Note that the maximal measurement error of 7.76% in [7] is valid here too, as the measured oblique shock wave cone angle is the basis for comparison to the simulated shock wave cone angle.

The notion that the phenomenon seen on NASA photo [6] is really a conical shock wave (and is therefore to be modelled as such) received confirmation by aerospace / aeronautics engineers [8].

And the modelled flow (fig. 1) is similar to the wind tunnel shadowgraph of a real *Saturn V* model [9] whose shock cone angle of 28° at Mach 1.93 correlates well with the 20.76° angle at Mach 2.9.

A 7-zip archive of the source database file for *Gambit* and case and data files for *Fluent* of the model is freely available for download and examination [10]. It is hereby declared "public domain".

## 4. Rocket velocity

Despite the correct Mach number, the rocket velocity calculation in [4] and [7] is wrong, as the retrorocket exhaust gases surrounding the rocket are much hotter than the counter air flow, which is not taken into account there. It is difficult to calculate the exact temperature of these gases (and thus the local speed of sound), but the rocket velocity can accurately be computed via the well-known Tsiolkovsky's formula substituting the mass ratio in it from the specific impulse formula [3]

$$I_{sp0} = \frac{F_0}{\dfrac{M_1 - M_2}{t_b}} \tag{1}$$

where $F_0$ is the sea level thrust, $I_{sp0}$ is the specific impulse at sea level, $t_b$ is the burn time, $M_1$ is the start mass, and $M_2$ is the mass at time $t_b$. As $F_0$ can be expressed as the product of the thrust-to-mass ratio (a.k.a. G-force) $Gf$ and the start mass $M_1$, formula (1) can be transformed as follows:

$$\frac{M_1 - M_2}{t_b} = \frac{F_0}{I_{sp0}}$$

$$\frac{Gf \cdot M_1}{I_{sp0}} = \frac{M_1 - M_2}{t_b}$$

$$\frac{Gf \cdot M_1 \cdot t_b}{I_{sp0}} = M_1 - M_2$$

$$M_2 = M_1 - \frac{Gf \cdot M_1 \cdot t_b}{I_{sp0}}$$

$$M_2 = M_1 \cdot \left(1 - \frac{Gf \cdot t_b}{I_{sp0}}\right)$$

$$\frac{M_2}{M_1} = 1 - \frac{Gf \cdot t_b}{I_{sp0}}$$

$$\frac{M_1}{M_2} = \frac{1}{1 - \dfrac{Gf \cdot t_b}{I_{sp0}}} \tag{2}$$

Substituting the mass ratio (2) in the Tsiolkovsky's formula [11] below, we get (3):

$$V_{ch} = g_0 \cdot I_{sp} \cdot \ln\left(\frac{M_1}{M_2}\right)$$

$$V_{ch} = g_0 \cdot I_{sp} \cdot \ln\left(\frac{1}{1 - \dfrac{Gf \cdot t_b}{I_{sp0}}}\right) \tag{3}$$

where $V_{ch}$ is the characteristic velocity, $g_0$ is the Earth's gravity, and $I_{sp}$ is the velocity-integrated mean specific impulse (slightly less than the specific impulse in vacuum). Substituting in (3) the known $g_0 = 9.80665$ m/s$^2$ [2], $Gf = 1.195$ [12], $t_b = 161.63 - 0.3 - (161.63 - 135.2) / 5 = 156.044$ s [14] (adjusted for all the five engines working all the time), $I_{sp} = 304$ s, and $I_{sp0} = 265$ s [13], we get:

$$V_{ch} = 9.80665 \times 304 \times \ln\left(\frac{1}{1 - \dfrac{1.195 \times 156.044}{265}}\right) = 3626 \text{ m/s} \qquad (4)$$

Subtracting the gravitational (1180) and aerodynamic (47) losses computed with [15], we get 3626 − 1180 − 47 = 2399 m/s. This is just 0.24% higher than the declared value of 2393.3 m/s [14]. (Interested readers can refer to Braeunig's refined and detailed *SA-506 / Apollo 11* simulation [16].) Note that no value substituted above could differ noticeably. E.g. if $Gf$ differs, the time the rocket clears the umbilical tower would differ, which will be noticed; same for $t_b$. And if any other but the central engine is cut off, flight trajectory control by gimballing the working engines could not work. Single-sequence footage [17] confirms the *Mach 1*, central engine cut-off, and staging times in [14].

## 5. Conclusion

A 3D computational fluid dynamics model of the *Saturn V* rocket has been created. The goniometric algorithm for rocket Mach number computation defined by the author in [7] has been verified by it and found to be exact within 1%. This simulation gave a more exact real Mach number of 2.9. However, the temperature of the retrorocket exhaust gases surrounding the rocket at the moment the photo [6] was taken was much higher, and so was the rocket speed. So, Boeing's value of ≈2.4 km/s [14] is correct, whereas the low velocity values in [4] by Pokrovsky and Popov and in [7] by the author are wrong, which this paper verifies via Tsiolkovsky's and specific impulse formulae.

## 6. Literature


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